Optimal Harvest and Forest Rotation Cycles

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Optimal harvesting in forestry is not quite the nice optimization you find in many applications. There is not a single simple curve that describes the peak. We cannot just use the first derivative test to find the maximum. We have to think a bit more carefully and we have to think about what the final goal really is.

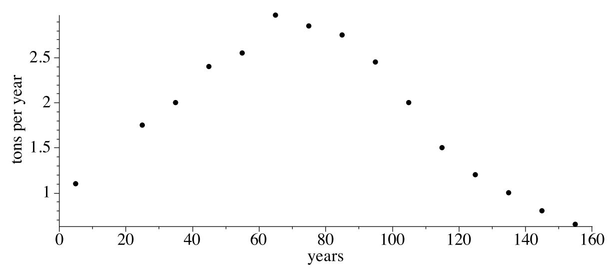
So we plan out what our goal is. The standard approach is to envision a maximum in the quantity of wood as the goal. To do this, we need to have an idea of the growth rate of the wood. This varies from species to species, tree to tree, and location to location. We will ignore variation within a stand and use data from a particular species from a particular forest.

Mountain Pond.		Black Walnut		Loblolly Pine	
time	rate	time	rate	time	rate
5	1.1	5	1.2	5	1.6
15	1.3	15	1.3	15	4.3
25	1.75	25	1.35	25	2.5
35	2.0	35	1.3	35	1.8
45	2.4	45	1.25	45	1.55
55	2.55	55	1.2	55	1.4
65	2.97	65	1.15	65	0.65
75	2.85	75	1.02	75	0.57
85	2.75	85	0.98	85	0.45
95	2.45	95	0.9	95	0.37
105	2.0	105	0.85	105	0.32
115	1.5	115	0.77	115	0.28
125	1.2	125	0.68	125	0.24
135	1.0	135	0.59	135	0.2
145	0.8	145	0.5	145	0.15
155	0.65	155	0.4	155	0.1

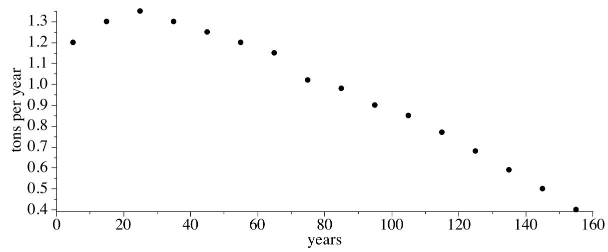
Table 1: Yearly growth rate as a function of time. Data taken from Richards (1993)

From this data, we get an idea of the shape of the curve and some of the tendencies in the grwoth of the trees. A plot for each of the species can also be of some help.

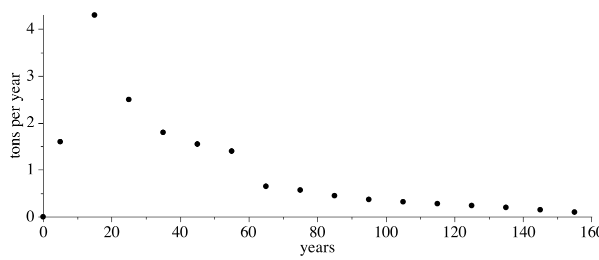
We start to see some generic features of the growth, such as a peak in growth that is followed by declining growth rates and perhaps asymptote to zero. We say perhaps because there is no data beyond 155 years. Since the peak is substantially higher than the growth rate near 150 years, we are probably safe



(a) Growth rate of mountain ponderosa pine



(b) Growth rate of Black Walnut



(c) Growth rate of loblolly pine

in assuming that a managed forest is not likely to be on a 150 (or longer) rotation cycle.

We also keep in mind that we are assuming, for now, a managed forest. We assume that the forest will be harvested. We can try to incorporate some ideas about the value of the ecosystem services later after we better understand the harvesting aspect.

The harvest time for maximum wood production occurs when the average of the growth rate from time zero to the present time is at a maximum. To find this, we need to be able to find the average growth rate. fitting the data to a function is essential although there are several ways to do this. We have used more than one method depending on the data. Splines work well. A general form where parameters are fit by numerical search also works well if you choose the functional form wisely (or with luck). For the data here we used Berkeley

Madonna to fit the parameters of the curve for Mountain Ponderosa. For the others, we used the spline command in Maple.
The average values come from the standard average value integral,

$$\frac{1}{T}\int_0^T f(x)dx,$$

and we are interested in the maximum value of this function. If you take the derivative and set it equal to zero, you find that the maximum occurs when the instantaneous growth rate is equal to the average growth rate. This happens in two place, at time zero and at the maximum.

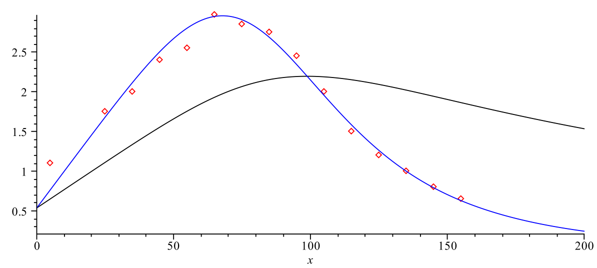


Figure 1: The blue curve is the fit of the data to a Hill function and the black line is the average from time 0 to the present time.

Because the curve is fit to data and is not an idyllic function, we look for a numerical optimum rather than an analytic one. We can either zoom in graphically or compute several values near the peak. It is also useful to note that since the data is reported on a yearly basis, our solution should not be more accurate than the nearest year.

Unfortunately this is not the end of the story because the goal is not really to grow the most wood, even if you are an environmentalist. There are really two goals depending on who you represent. First, you might be the land owner or lumber company and what you really care about is money. So the real question then is what should the rotation cycle be if you want to maximize your profits? The crucial thing here is to note that on average wood products from larger trees are worth more per ton of wood than products from smaller trees. So there is an monetary advantage to selling larger trees than smaller ones. How much of a difference that might make and how much older the trees should be is the issue we would like to address.

But it is not really growth that we want to optimize, it is money. this might be different. We found some data on the product distribution (generic pine) as a function of the age of the tree and fit a series of functions to it. The formatting leaves a bit to be desired, but that is related to the way we constructed the function.

$$Logwood = Heaviside(HP-40)\frac{(0.6*(HP-40)^4}{(30^4+(HP-40)^4))}$$

$$Pulpwood = 0.85 + Heaviside(HP-40)(-.55 + \frac{.55}{(1+\frac{.25}{(30^4*.3)}(HP-40)^4))}$$

$$WasteBarkFuel = 1 - logwood(HP) - pulpwood(HP)$$

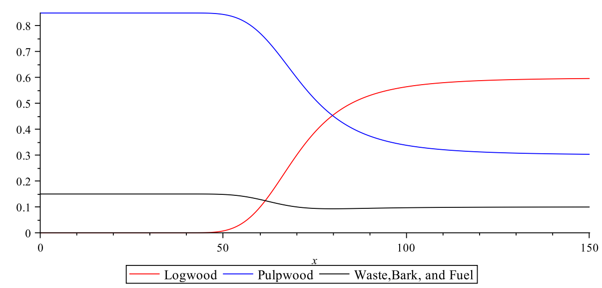


Figure 2: Product distribution in fractions as a function of the age of the tree. Data from Kaipainen et al (2004)

The actual prices for the different classes of wood products changes over time and depends on many different factors. For this exercise, we will assume different values and look at the trend and magnitude of the change in the optimal value. For a proof of the general tendency and conditions on the existence of the maximum, you will have to wait a few months until the paper is published ...
There is an additional factor that has not been taken into account as well. As climate change treaties have developed and changed over the last few years, it has become clear that some sort of carbon market is likely. The primary cause of this is the recognition that there is a cost (monetary value) to the release of carbon to the atmosphere. Exactly how those payments are structured and who collects such a payment has not been worked out. However once such a cost is formally recognized, it is a short step to suggest that the costs of carbon release must be paid at the time of release. For wood products, this means that longer lived products need not be paid for when they are produced but when their useful lives are at an end. Since the value of money now is not necessarily the same as the value of a future stream of money because of discounting effects, the carbon released from long lived products may be valued substantially less than a shorter lived product. As it turns out, larger trees tend to produce longer lived products. This means that the present value of carbon released from longer lived products is less than the present value of carbon released from shorter lived products. We can incorporate this into our scheme above as well.

It turns out that the tendency for a reduced present value of emissions is equivalent to a larger income from the sale of a product. This means that the effect on the optimal rotation cycle of the harvested trees is similar for taking into account product pricing and carbon payments. We will leave the details for the published paper.

As an exercise, determine the optimal rotation cycle for the three species of wood. Then investigate the impact of the product distribution and pricing of the wood products that come from the harvest.

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